A Shear Transfer Model for the Lincoln Block® System

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1 Goals

The goal of this report is to analyze the in-plane shear transfer of the Lincoln Block® System as is currently recommended. We supplement the July 30th 2019 Nickerson Engineering preliminary structural calculations for the 33 × 25 cabin by considering the effects of the polyurethane foam, glue, and silicone selant. These additional factors set the members' axes of rotation and determine the distribution of the wall's internal forces. Once a shear modulus for pure block is acquired, we use energy arguments to approximate the additional strength added by the required corner pieces and splines for any given Lincoln Block® shear wall. In section 3.5 we apply these equations to the specifics of the 33 × 25 cabin.

These results are preliminary and we seek stakeholders and interested parties to help provide experimental validation of the model's components. Values in this paper reflect the physics alone; ASD or LFRD adjustments given design factors appropriate to project specifics are advised.

2 Mid-wall Shear Transfer

2.1 Parameters

Shear transfer between courses of Lincoln Block® is enabled by regularly spaced members in the cavity and a pair of tongue and grooves on the sides filled with an elastometric sealant. The members are secured to their home course by four nails and two patches of elastometric adhesive weeks before construction. At the build site, courses are stacked such that these members fit snugly into the course above and are subsequently nailed in place. The resulting cavity is then filled with spray polyurethane foam (SPF).

The reaction forces and torques of these members depend on the relative course drift Δx, and both the members' axis height h and rotation angle $\Delta\theta$ (described in figure 1). Heights in this section are assumed to be measured in inches above the bottom of the course under consideration.

After analyzing block-to-block shear connections, we develop a model for shear transfer that includes the corner pieces and splines which facilitate point loads and tie each course, in a one-story Lincoln Block® structure, directly to the sill plate and foundation.

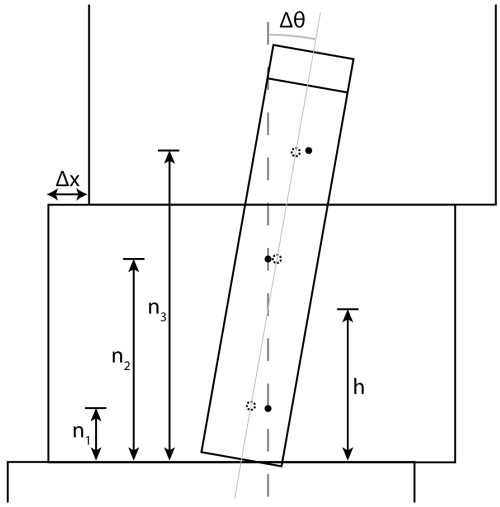

Figure 1: Side view of course-to-course connection showing the coordinate system chosen to describe the rotation/translation of the members.

2.2 Yield Conditions

The proposed mode of shear failure for Lincoln Block® is via excessive bending of the brad nails that secure the sides to the members. Such nails, when forced opposite their bend, lever and aid in the over-extension of the elastometric adhesive, which may lead to the eventual disunion of the sides and members. The design of the Lincoln Block® system aims to prevent this eventuality with adhesive SPF in the cavity and rugged spline pieces nailed inside the walls at critical junctures.

2.3 Components

2.3.1 Industrial Sealant

Standard Lincoln Block® features a tapered tongue and a concentric groove. This allows for the encasement of a 1.01 *in* wide, 0.06 *in* thick bead of opaque DAP Dynaflex 230 sealant. Joint movement and elongation at break are listed by the manufacturer as 25% and 300% respectively[5]. Thus under a horizontal drift of $0.06\sqrt{1.25^2-1}\ in = 0.045\ in$ the bead is stretched to 125% of its initial length while the sealant's maximum drift, $0.06\sqrt{4^2-1}\ in = 0.23\ in$, is far more than the course drift required for the first nail to yield.

The force required to cause a specific drift is determined by the shear modulus of the sealant itself however this information is not available publicly. As stress-strain curves are typically concave down, we *underestimate* the elastic modulus of the sealant as the secant modulus at maximum strain and ultimate strength.

Dynaflex 230 has an ultimate tensile strength of 200 *psi* at 300% elongation hence we estimate the elastic modulus as $E_s = 66.7\ psi$. Given a Poisson ratio of $\nu_s = 0.4$ we approximate the shear modulus of this isotropic sealant G_s as

$$G_s = \frac{E_s}{2(1+\nu_s)} = 23.8\ psi$$

Thus we assume one 24 *in* bead acts with a restoring force

$$F_s/\Delta x = \frac{23.8\ psi \cdot (1.01 \cdot 24\ in^2)}{0.06\ in} = 9615\ lbf/in$$

near equilibrium.

2.3.2 Members and Nails

The members of the Lincoln Block® system are designed to be 3.125 *in* wide and 7.625 *in* high, and are regularly spaced 24 *in* o.c. Members are nailed to their home course at 1 *in* and 3.75 *in* by two pairs of Grip Rite 0.072 $in \times 2\ in$ stainless steel brad nails and glued with two patches of Sikaflex 1A adhesive. When constructed, another pair identical nails at 5.75 *in* secures the block, 1 *in* into the adjacent course.

Observing in-house experiments, nail heads do not typically pull through before the shanks pull out of the member, and only after significant bending and yielding of the wood has occurred. By employing the American Wood Council's equations[1], we determine the yield mode for these nails given the following parameters:

- Specific gravity of Douglas fir $G = 0.46$
- Nail diameter of $D = 0.072\ in$
- Dowel bearing resistance $q = 16,600DG^{1.84}$
- Nail yield bearing of $F_{yb} = 169,000 - 278,000D$
- Nail bending moment of $M_d = \frac{F_{yb}D^3}{6}$
- Side-member gap, $g \approx 1/64\ in$
- Nail embedment depths
  1. $L_s = 1.125\ in$
  2. $L_m = 0.859\ in = 2\ in - L_s - g$.

Here L_s and L_m are nail lengths in the side and member respectively as the nail gun typically overdrives nails 1/4 *in* beyond flush. Via the General Dowel Equations for Solid Cross Section Members, we find that bending in which two plastic hinges form (mode IV) is the preferred mode of nail yielding.

Mode	Yield Force (lb)
I_m	246.1
I_s	322.2
II	83.11
III_m	90.96
III_s	113.97
IV	<u>70.78</u>

To determine the restoring shear of individual nails one can calculate the slip modulus of a single such connection analytically. However, section 7.1 of Eurocode 5[6] claims the following empirical two-parameter power law model is sufficient for non-pre-drilled timber-to-timber connections:

$$k = \rho^{1.5}d^{0.8}/30 \tag{1}$$

where ρ is the density of our timber in kg/m^3, and d the dowel diameter in mm. Using $1000kg/m^3$ times the specific gravity of Douglas fir and converting our dowel diameter into appropriate units, we have

$$k = 533.0\ N/mm = 36520\ lbf/in.$$

Using our choice of coordinates (in figure 1) and given a course drift of Δx, the force applied by each nail on the member is given by,

$$F_{n_1}(\Delta x, h) = -k\Delta\theta(\Delta x, h) \cdot (n_1 - h)$$
$$F_{n_2}(\Delta x, h) = -k\Delta\theta(\Delta x, h) \cdot (n_2 - h)$$
$$F_{n_3}(\Delta x, h) = -k\left[\Delta\theta(\Delta x, h) \cdot (n_3 - h) - \Delta x\right]$$

where n_1, n_2, and n_3 are the nail positions 1, 3.75, and 5.75 *in* while $\Delta\theta(\Delta x, h)$ is to be determined by equilibrium conditions.

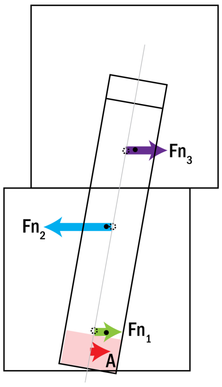

Figure 2: Schematic diagram of forces by nail and adhesive on member for some h, $\Delta\theta$, and Δx.

Later, we consider the torque by each nail about an axis at height h,

$$\tau_{n_i}(\Delta x, h) = F_{n_i}(\Delta x, h) \cdot (n_i - h)$$

and then use our zero net-torque condition in order to determine the preferred axis of rotation.

2.3.3 Elastometric Adhesive

In the construction of Lincoln Block®, an elastometric adhesive (currently Sikaflex 1A) is applied to the members before being nailed together and left to cure. The torque and restoring force of the adhesive is a function of the adhesive thickness, axis of rotation, and angle of rotation.

Assuming the incomprehensibility ($\nu_A = 0.5$) of Sikaflex 1A we have a shear modulus of $G_A = 35/3\,psi$ (at up to 25% extension), an axis at height h, adhesive thickness t, and assuming complete and uniform coverage we find the restoring torque of the adhesive, τ_A, about a central axis $(0.75, h)$ as follows.

$$\begin{aligned} G &= \frac{dF/dA}{r\Delta\theta/t} \\ dF &= \frac{G\Delta\theta}{t} r dA \\ d\tau_A &= -r \cdot dF \\ \tau_A &= -\frac{G_A\Delta\theta}{t}\int_0^{4.75}\int_0^{1.5}(x-0.75)^2 + (y-h)^2 dx dy \\ &= -\frac{G_A\Delta\theta}{t} 0.59375\left(12(h-2.375)^2 + 24.8125\right) \end{aligned}$$

where $r = \sqrt{(x-0.75)^2 + (y-h)^2}$ and the negative sign ensures τ_A acts opposite $\Delta\theta$.

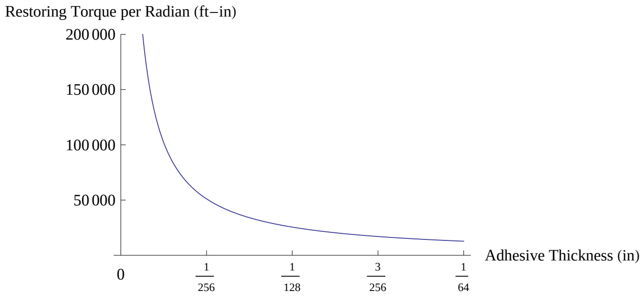

Figure 3: Restoring torque of Sikaflex 1A adhesive given a 1.8 in high axis showing resistance is inversely proportional to the thickness.

By symmetry, the Sikaflex adhesive supplies no net-force in our model when $h = 2.375\ in$. For all other axes there exists a patch of adhesive below/above the axis for which there is no patch above/below to counter act. This force acts horizontally and is given by summing up the horizontal component of the adhesive's restoring force,

$$F_A = \frac{G\Delta\theta}{t}\int_0^{1.5}\int_0^{4.75}(h-y) dy dx.$$

Adhesive thickness is determined via measurement of the sides, member, and total width of prepared Lincoln Block®. Total width is taken as the average of the top and bottom widths.

Side 1	Side 2	Member	Width	Glue
1.401	1.395	3.141	5.9615	0.012
1.408	1.395	3.134	5.9865	0.019
1.407	1.393	3.130	5.974	0.006
1.397	1.389	3.137	5.953	0.002
1.413	1.392	3.124	5.968	0.015
1.400	1.400	3.130	5.9595	0.012
1.390	1.395	3.136	5.9395	0.008
1.396	1.396	3.148	5.9475	0.004
1.394	1.400	3.146	5.9585	0.005
1.401	1.395	3.136	5.961	0.009

Table 1: Measurements of manufactured Lincoln Block ® using digital calipers. All values are in inches

The mean and standard deviation give us an estimated adhesive thickness of $0.009 \pm 0.005\ in$. As $1/64 = 0.015625$ we maintain it as an over estimate of the gap and adhesive thickness. Stiffer adhesives with minimal thickness should be used to discourage member rotation.

2.3.4 Polyurethane Foam

Lincoln Block® is designed with sufficient strength to accommodate the usage of DAP Touch'n Seal, a fire retardant spray polyurethane foam (SPF), sealed inside its cavity. Besides serving as insulation, this foam adheres to the sides and members increasing the internal cohesion of the block system, allegedly acting as a fail safe to prevent the disunion of the sides from each other and the members.

In this analysis the foam applies no net force on the member by symmetry. This assumption is violated if adjacent courses are displaced different distances, or if members are irregularly spaced. However, the forces by the foam are several times smaller than the forces by the nails so we maintain this assumption for simplicity.

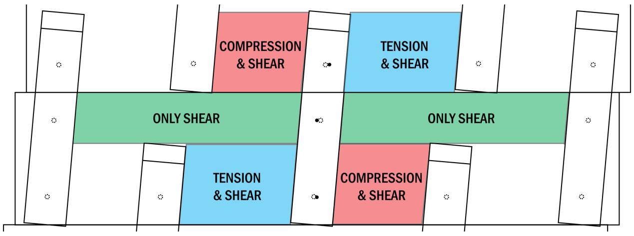

Figure 4: Forces considered for each rectangular block of foam adjacent to a given member.

The data on shear and compressive modulii of DAP are proprietary, however ultimate strengths and elongations at break are provided[5]. Note elongation for compressive failure was not given, and is hence *assumed identical* to tensile elongation at break.

	Ultimate Strength	Elongation @ Break	Secant Modulus
Compressive	11 psi	10%	110 psi
Tensile	33 psi	10%	330 psi
Shear	18 psi	57%	31.5 psi

Table 2: Details of modulii estimates for DAP SPF.

Members penetrate 2.875 *in* into neighboring courses allowing for nailing. Assuming the member in question (1.5 *in* wide) is halfway between the two adjacent members, there is a 10.5 *in* long block of foam on either side of the top and bottom of a mid-wall member. As the force on the member by the tension and compression act in the same direction by similar length blocks, we treat them as a single block with elastic modulus 440 *psi*. As the patch width varies from 3.125 *in* wide down to 1.5 *in* in the top 5/8 *in* of each member we integrate to find the torque about an axis h due to the top and bottom foam as

$$\begin{aligned}\tau_{f1} = &-\frac{440\Delta x}{6.5}\Big(\int_{4.75}^{7} 3.125(z-h)dz \\ &+\int_{7}^{7.875}(3.125-2.6(z-7))(z-h)dz\Big)\\ \tau_{f2} = &-\frac{440\Delta x}{6.5}\Big(\int_{0}^{2.25} 3.125(z-h)dz \\ &+\int_{2.25}^{2.875}(3.125-2.6(z-2.25))(z-h)dz\Big).\end{aligned}$$

Restoring shears are calculated in less detail ignoring the tapered geometry of the members. Given $G_f = 31.5$ *psi* we find the shear torques starting from the bottom at heights 0 *in*, 2.875 *in*, 4.75 *in*, and 7.875 *in* corresponding to the boundary of the three regions described in figure 4. The shear torques acting at each surface relative to an axis h is given by,

$$\begin{aligned}\tau_0 &= -G_f \cdot 3.125(10.5)(h-0)\Delta\theta\\ \tau_{2.875} &= -G_f \cdot 3.125(11.25)(h-2.875)\Delta\theta\\ \tau_{4.75} &= -G_f \cdot 3.125(11.25)(h-4.75)\Delta\theta\\ \tau_{7.875} &= -G_f \cdot 3.125(10.5)(h-7.875)\Delta\theta.\end{aligned}$$

where 10.5 *in* and 11.25 *in* correspond to the local lengths of foam considered to act on this member (given that neighboring members share the shear force).

In this model, the foam only influences the axis of rotation of the members. We conservatively neglect the force needed to deform the foam itself in the analysis of course-to-course connections for simplicity in this model.

2.4 Equilibrium Conditions

In our quasi-static analysis we consider adjacent courses of Lincoln Block® held at maximum extension. We solve for $\Delta\theta(\Delta x, h)$ by enforcing zero net-force on the members, then for h by enforcing no net-torque, and finally the maximum Δx is determined by enforcing that no nail joint experiences forces in excess of 70.78 lbf. This gives

$$\Delta\theta(\Delta x, h) = \frac{0.3179\Delta x}{h-3.448} \qquad (2)$$

in radians, with a vertical asymptote at $h = 3.448$ *in*. As we have used the small angle approximation throughout, this formula is a valid approximation when $|\Delta\theta| \lesssim 0.09$ radians, or about 5^o. Axes near 3.448 *in* require angles in excess of this limit. However such states require external torques to maintain and hence they do not occur.

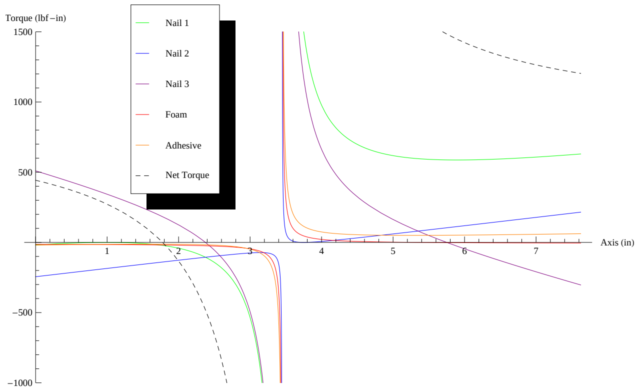

Figure 5: Torque due to each component and net torque as a function of rotation axis.

We find an axis of $h = 1.776$ in is the only solution corresponding to no net force and no net torque on the members. Increasing Δx, the first nail yields when $\Delta x = 0.00516$ in as represented in figure 6.

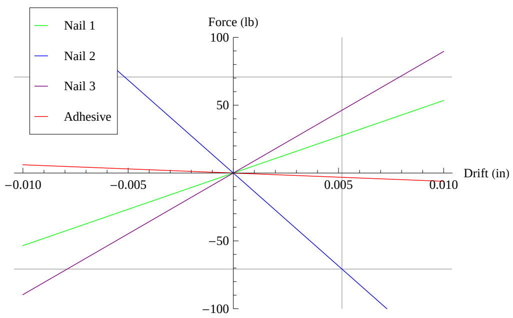

Figure 6: Force displacement functions for the given axis with the horizontal lines at ± 70.78 lbf representing the threshold force for mode IV bending.

Evaluating each function at for $\Delta x = 0.00516$ in gives us the following force distribution:

Component	Disp. (in)	Force (lb)	Var.
Nail 1	-0.00076	$+27.8$	Fn_1
Nail 2	$+0.00194$	-70.8	Fn_2
Nail 3	-0.00126	$+46.1$	Fn_3
Adhesive	$+0.00233$	-3.13	F_A
Sealant	∓ 0.00516	± 49.7	F_S

Table 3: Displacement and force of each component at maximum extension with a DAP foam filled cavity.

The sealant force here is that of a single 24 in bead, corresponding to the ± 12 in neighborhood of a single member.

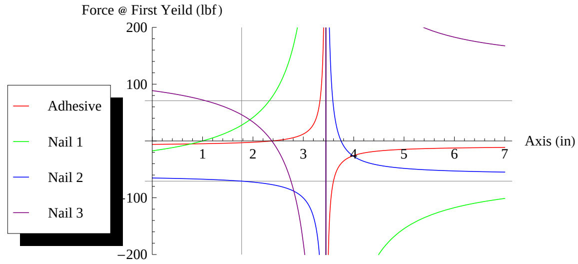

Figure 7: Forces by nails and adhesive as function of axis given $\Delta x = 0.00516$ in. The vertical line corresponds to the axis $h = 1.776$ in.

Considering now the forces on two adjacent courses we produce an estimate of the shear strength of 24 in of un-aided Lincoln Block® as $2(Fn_1+Fn_2+F_A+F_S)$ or equivalently $2(Fn_3+F_S)$ each of which give 191.5 lbf.

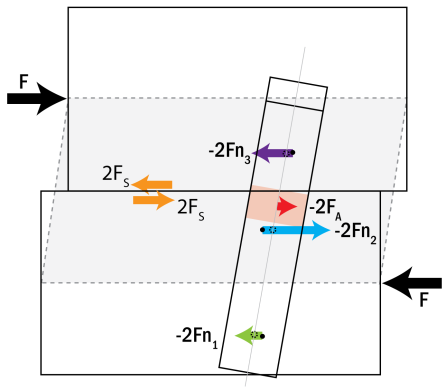

Figure 8: Forces on the sides due to nails in member (purple/blue/green), adhesive (red), and sealant (orange).

We would be remiss to neglect the case in which no spray foam is used. Using the same arithmetic we neglect the torques and forces described in section 2.3.4 and solve again for the axis corresponding to equilibrium. This yields $h = 1.811$ in, a virtually identical max extension of $\Delta x = 0.00515$ in, and a maximum shear transfer of 187.3 lbf, only about 4 lbf less than a similar 2-foot wall filled with SPF. This may be partially caused by our underestimates of the spray foam's rigidity in section 2.3.4. The resulting force distribution is included below in table 4.

Component	Disp. (in)	Force (lb)	Var.
Nail 1	-0.00081	$+29.6$	Fn_1
Nail 2	$+0.00194$	-70.8	Fn_2
Nail 3	-0.00121	$+44.2$	Fn_3
Adhesive	$+0.00237$	-3.00	F_A
Sealant	∓ 0.00515	± 49.5	F_S

Table 4: Displacement and force of each component at maximum extension given an empty cavity wall.

From here on we follow Lincoln Block® recommendations and consider only walls filled with DAP SPF. Dividing the 191.5 lbf value from before by the conversion factor 24/12 we find that un-reinforced block should transfer **95.75 lbf** per lineal foot of block at a course drift of a mere 0.00516 in. This yields a linear force law for use elsewhere in this model. Alternatively we can encapsulate this relation as the shear modulus of a "virtual block" (e.g. the dashed parallelogram in figure 8) between adjacent course centers as,

$$G_{block} = \frac{191.5 \; lbf/(24.0 \cdot 5.96 \; in^2)}{0.00516 \; in/4.75 \; in} = 1230 \; psi.$$

3 The System

We emphasize at this point Lincoln Block® is a *system*. The above calculations only apply to block that has not been reinforced by spline pieces. Never would such a wall be built. At minimum, each corner would have a glue laminated 3.125 in^2 beam nailed to each block face by pairs of 16d, 0.135 $in \times 3.5$ in nails. Further, 3.125 $in \times 1.5$ in spline pieces secured with 0.090 $in \times 2.5$ in ring-shank nails are added to support various point-loads, and to stiffen the wall around window frames and doors[10].

In order to combine these elements we calculate the work required to bend/shear the splines/corners pieces, to deform their nails, and to translate each attached course. We accomplish this by assuming an appropriate deflection profile based on the loading mode. Using the slope of the profile's tangent at each interface we determine the relative course drift for each pair of adjacent courses.

3.1 Splines and Corners

We begin by determining the energy required to bend the framing elements of a Lincoln Block® wall. As the first course is tied to the foundation and floor directly with HTT4 ties anchoring each corner to the sill plate, we enforce boundary conditions such that both splines and corners meet the sill plate perpendicularly. This allows us to model point loading (1) or uniform loading (2) of these beams and find the associated deflection profiles $\omega(z)$. These profiles are assumed to govern the shape of a deformed wall. Point loading refers to a concentrated force F applied at the top of the wall and the associated reaction $-F$ at the foundation. This is a static load designed to mimic seismic loading. The uniform loading however is analogous to a wind load, applied to an adjacent wall and transferred via normal forces to the shear wall in question via the corner assembly.

Let n be the number of courses and c be the course height. Given the yield conditions developed in section 2, we determine the profile by fixing $\omega'(H) = \Delta x/c$ where $n \cdot c = H$ is the height of the beam and where the slope is greatest. This guarantees Δx is the maximum drift per course. It follows,

$$\omega_1(z) = \frac{1}{3}\frac{\Delta x}{c}\frac{z^2}{H^2}(3H - z)$$
$$\omega_2(z) = \frac{1}{4}\frac{\Delta x}{c}\frac{z^2}{H^3}\left(6H^2 - 4Hz + z^2\right)$$

where,

$$F = \frac{2EI}{H^2}\frac{\Delta x}{c} \text{ and } q = \frac{8EI}{H^3}\frac{\Delta x}{c}$$

are minimum force and linear pressure required to ensure the maximum slope above for single splines. These correspond to a max deflections of

$$\omega_1(H) = \frac{2}{3}n\Delta x \text{ and } \omega_2(H) = \frac{3}{4}n\Delta x.$$

An example of course drift profiles for a 21 course wall given the maximum Δx from section 2.4 is presented in figure 9.

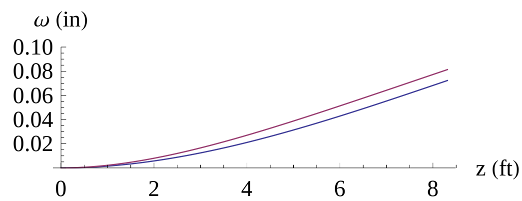

Figure 9: Bending profiles for an 8.3 ft beam with fixed max slope given point (blue) and uniform loading (burgundy).

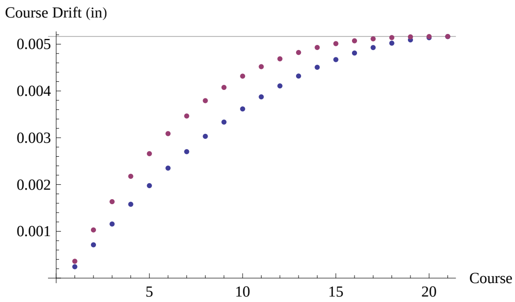

Figure 10: Plot of the relative course drift, $c\omega'(c \cdot i)$ for each loading (colored as in figure 9). Course drift is most pronounced high on the wall and approaches 0.00516 *in* in this example.

3.1.1 Bending

With these profiles in hand we calculate both the bending and shear energy in the beams. Let b and m be the dimensions of the beam cross section, where m is the width in the direction of the bending. The area moment of inertia of the cross section through its centroid is simply

$$I = \int_{-b/2}^{b/2} \int_{-m/2}^{m/2} x^2 dx dy = \frac{bm^3}{12}.$$

The corners pieces are a two-piece glulam beam, however as the neutral axis corresponds to the glue layer we use the above area moment of inertia nevertheless. Given the moment $M(z) = -EI\frac{d^2\omega}{dz^2}$ the bending energy is calculated via,

$$U_{bend} = \int_0^H \frac{M^2(z)}{2EI} dz.$$

Given point (1) and uniform loading (2) we have,

$$U_{1,bend} = \frac{1}{18}\left(\frac{\Delta x}{c}\right)^2 \frac{E}{H}(bm^3)$$
$$U_{2,bend} = \frac{3}{40}\left(\frac{\Delta x}{c}\right)^2 \frac{E}{H}(bm^3).$$

For splines $m = 1.5$ *in* and for corners $m = b = 3.125$ *in*. Given the elastic modulus of Douglas fir we have,

$$U_{iJ,bend}(\Delta x, n) = B_{iJ}\left(\frac{\Delta x^2}{n}\right)$$

where the coefficients B_{iJ}

B_{iJ}	$i = 1$	$i = 2$
S	1.0661×10^4	1.4393×10^4
C	9.6401×10^4	1.30140×10^5

allow the calculation of the energy required to bend each corner (C) and spline piece (S).

3.1.2 Shear

While bending energy dwarfs shear energy in thin beams, we calculate it nevertheless for completeness. For the point loading, the shear is constant while for uniform loading the shear force at a height z is due to the remaining uniform load above z, as the load below counteracts the reaction at the first course. This gives a shear function of

$$V(z) = q(H - z)$$

in the second case. The corresponding energy is given by,

$$U_{shear} = f_s \int_0^H \frac{V^2(z)}{2GA} dz$$

where the rectangular beam shape factor is $f_s = 6/5$. This yields,

$$U_{1,shear} = \frac{12}{5}\left(\frac{\Delta x}{c}\right)^2 \frac{E^2 bm^5}{GH^3}$$
$$U_{2,shear} = \frac{1}{20}\left(\frac{\Delta x}{c}\right)^2 \frac{E^2 bm^5}{GH^3}$$

which again differ only by a constant factor. After substituting $A = bm$, $H = nc$, and q from above,

$$U_{iJ,shear}(\Delta x, n) = S_{iJ}\left(\frac{\Delta x^2}{n^3}\right)$$

where the coefficient S_{iJ} is given by the following table.

S_{iJ}	$i = 1$	$i = 2$
S	6.5612×10^6	1.3669×10^5
C	2.5750×10^8	5.3646×10^6

This term dominates for short beams but diminishes for longer beams given the factor of n^3 corresponding to the wall height.

3.1.3 Additional Nails

Corner pieces and splines are typically secured with nails 1 *in* from course edges. Repeating the same calculations assuming flush nailing without gaps for these additional nail types we find the preferred yield mode and force.

Mode	Ring-shank (lbf)	16d (lbf)
I_m	403	1369
I_s	492	886
II	187	485
III_m	149	397
III_s	177	356
IV	<u>112</u>	<u>337</u>

Again following the Eurocode 5 empirical relation (equation 1) for these Douglas fir timber-to-timber nail joints we calculate

$$k_{16d} = 637.2\ N/mm = 43662\ lbf/in \text{ and}$$
$$k_{rs} = 881.4\ N/mm = 60392\ lbf/in$$

to be the prescribed slip modulii for these nails.

We now consider the work that must be done in elastic deformation these nails. Using the slope of the profile $\omega'(z)$ we capture how the bending of each nail varies based on the angle of the beam at that course. Assuming that the courses track the beam and that the axis locally is equidistant from each nail, we *underestimate* the work needed to deform the pair of nails. This also implicitly allows courses to drift without applying a net force on the splines or corner pieces. Hence the approximate shear displacement for any nail is

$$\Delta = (c/2 - 1) \cdot \omega'(c(i - 1/2)) \ : \ 1 \leq i \leq n$$

where $c = 4.75\ in$ is the course height.

Hence the work required to bend a pair of the 16d nails is

$$U_{16d,i} = 2 \cdot \frac{1}{2} k_{16d} \Delta^2$$

while to bend four 0.090 in ring-shank nails the work required is

$$U_{rs,i} = 4 \cdot \frac{1}{2} k_{rs} \Delta^2.$$

Thus, given the appropriate profiles, the energies required to bend all of the nails securing the corners and splines are

$$\begin{aligned} U_{1C,nails} &= \sum_{i=1}^{n} U_{16d,i} \\ &= \frac{21.0856\left(7 + 128n^4\right)}{n^3} \\ &\approx 2699.0(\Delta x^2 n) \text{ and} \\ U_{1S,nails} &= \sum_{i=1}^{n} U_{rs,i} \\ &= \frac{30.4889\left(7 + 128n^4\right)}{n^3} \\ &\approx 3902.6(\Delta x^2 n) \end{aligned}$$

for point loading, and

$$\begin{aligned} U_{2C,nails} &= \frac{3.765\Delta x^2}{n^5}\left(864n^6 + 196n^2 - 31\right) \\ &\approx 3253.2(\Delta x^2 n) \text{ and} \\ U_{2S,nails} &= \frac{5.444\Delta x^2}{n^5}\left(864n^6 + 196n^2 - 31\right) \\ &\approx 4704.0(\Delta x^2 n) \end{aligned}$$

for uniform loading. We use the exact expressions in our analysis with these approximations provided only for comparison.

3.2 Block

Pulling from our earlier work, we deduced that foamed courses of block have a shear modulus of $G_{block} = 1509\ psi$. Given a course drift of Δx_i the work to shear a adjacent courses of block is

$$U_{B,i} = \frac{1}{2}\frac{GA\Delta x_i^2}{c},$$

where $\Delta x_i \approx c\omega'(c \cdot i)$ from the slope of the profile. Hence, for one foot of block, n courses high,

$$U_B = \frac{1}{2} GAc \sum_{i=1}^{n-1} \omega'(c \cdot i)^2$$

Given $A = 12 \cdot 5.96\ in^2$, and the corresponding ω' this yields,

$$\begin{aligned} U_{1B} &= \frac{309(n-1)\left(16n^3 + n^2 + n + 1\right)\Delta x^2}{n^3} \\ &\approx 4947(n-1)\Delta x^2 \\ U_{2B} &= \frac{221(n-1)(27n^5 + 6(n^4 + n^3 + n^2) - n - 1)\Delta x^2}{n^5} \\ &\approx 5963(n-1)\Delta x^2 \end{aligned}$$

3.3 External Work

In the following section we proceed to calculate Δx (the max per course drift) given a fixed point or uniform load. These same calculations will also allow us to determine what external loading causes threshold course drift. In order to do either we must first find the external work done on a wall, length L feet, and n courses high (4.75 in per course).

For point loading, the expression

$$U_{1,ext} = \frac{Fn\Delta x}{3}$$

is sufficient assuming a linear response by the wall. For uniform loading, given the profile $\omega_2(z)$ the external work done in deforming the wall section is,

$$\begin{aligned} U_{2,ext} = \int_0^H \frac{1}{2} Q\omega_2(z) dz &= \frac{3}{20}\left(\frac{\Delta x}{c}\right) QH^2 \\ &= \frac{3Qn^2 c\Delta x}{20}. \end{aligned}$$

3.4 Results and Analysis

Let δ be the density of splines (in splines per foot) and suppose there are two corner pieces. Assuming that all work done on the wall is transformed into the potential energy of its components we solve

$$\begin{aligned} U_{ext} = & 2(U_{C,bend} + U_{C,shear} + U_{C,nails}) \\ & + \lfloor L\delta \rfloor (U_{S,bend} + U_{S,shear} + U_{S,nails}) \\ & + L \cdot U_B \end{aligned} \quad (3)$$

for Δx and alternatively for F or Q as a functions of n, δ, L, and the remaining variable. Recall the floor function $\lfloor \cdot \rfloor$ returns the largest integer less than or equal to the argument.

3.4.1 Point Loading (Seismic)

First we consider the maximum course drift Δx given a fixed load. Referring to the Nickerson Engineering preliminary calculations, a seismic base shear of 1895 lbf corresponds to an $R = 6.5$ earthquake given site conditions in Seismic Design Category D[7]. This gives a fixed seismic load of 948 lbf for each wall shear wall. We will use this fixed load multiple times, however the seismic base shear is a function of the building's mass, and this quantity in was computed specifically for the 33 $ft \times 25$ ft cabin. In light of this we also provide maximum projected (per wall) seismic base shear loads in figures 13, 14, and 15.

To proceed we fix F we solve for $\Delta x(n, L, \delta)$ as a function of the number of courses, wall length, and spline density. Spline spacing is irregular in practice and partial and mid-wall splines are often employed. Hence the density parameter δ is a proxy for the amount of reinforcing splines inserted into the wall. To establish lower bounds, one can set δ equal to zero.

For the sake of this analysis however we will routinely assume full height splines are conservatively placed every 12 feet. This allows us to plot the response of shear walls ranging from 5 to 25 feet long.

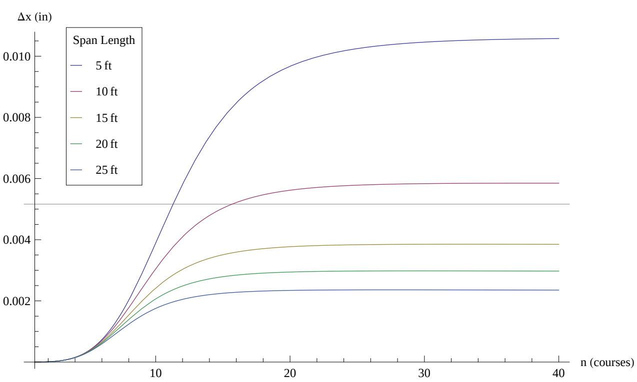

Figure 11: Max expected course drift under a fixed 948 lbf lateral point load for various length and height unperforated walls, reinforced with splines every 12 ft.

This model assumes *long* boards or tightly laid courses in which the normal force acts between blocks. However, gaps between blocks on the same course and spline deflection at the sill plate are neglected factors that could lead give rise to additional course drift. In practice these neglected gaps are about 0.02 in wide and reinforced with a short spline piece secured with $8-0.072$ $in \times 2.0$ in brad nails. For example, if on average 5 foot blocks are used along a 25 foot wall, these gaps total 0.08 in across the span.

This model confirms what contractors know from experience: additional splines surely must be added shorter walls to ensure rigidity. In figure 12, this is effect is illustrated for a short wall given various spline spacings, where the horizontal line corresponds to the course drift of first yielding.

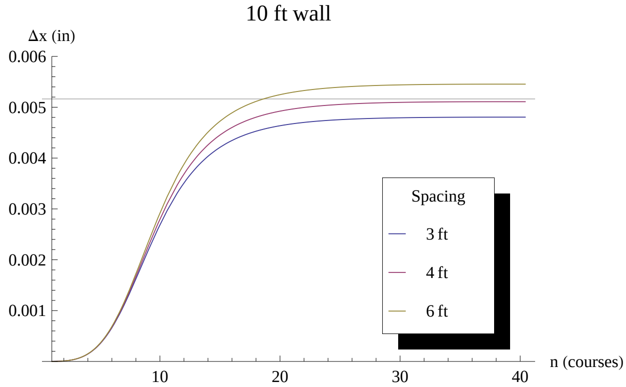

Figure 12: Max expected course drift for reinforced unperforated 10 ft long walls.

As with other building systems we see that short length walls of block cannot serve as shear walls, and require excess reinforcement to prevent over extension. One achieves these results via splines, reducing the required wall height, or by the inclusion of other longer parallel shear walls in the design.

While this analysis is useful we look to determine the threshold forces required to over-extend a Lincoln Block® wall of any length and height allowable with continuous splines (i.e. less than 24 ft high). Given $U_{1,ext} = \frac{Fn\Delta x}{3}$ we solve for F via equation (3) in section 3.4. Fixing $L = 25$ ft we produce the following chart as a function of the number of courses and spline spacing. For continuity we round down the floor function replacing $\lfloor L\delta \rfloor$ with $(L\delta - 1)$. Below in figures we scale this force by the wall length to give pounds-force per lineal foot shear wall.

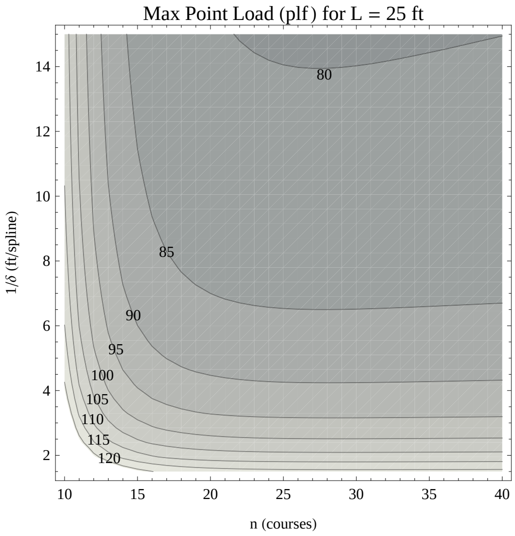

Figure 13: Max point loading for a 25 ft long wall.

Conversely, for a fixed spline density (e.g. 12 ft per spline) we produce a similar chart for height and length. The color scheme is inconsistent so please refer to the contour labels.

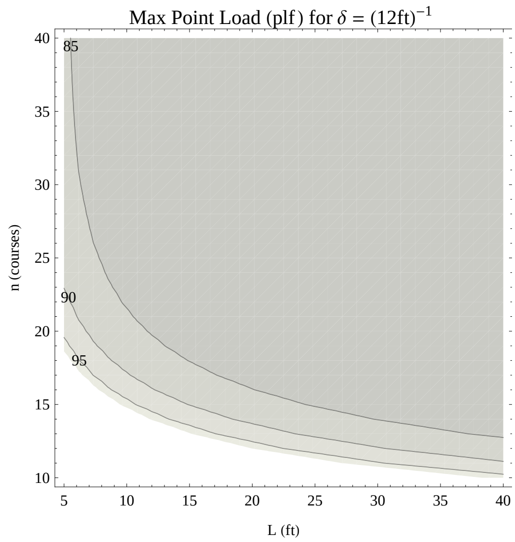

Figure 14: Max point loading for walls reinforced with splines every 12 ft.

Alternatively we represent the above result for $\delta = 1/12\ ft^{-1}$ in figure 15 for specific wall lengths.

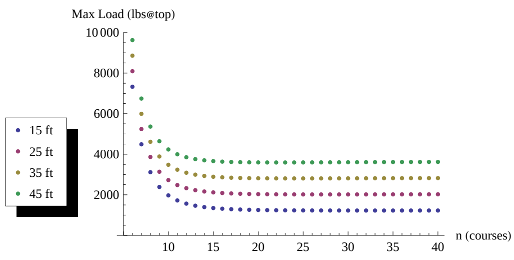

Figure 15: Alternative representation of max point loading for walls reinforced with splines every 12 ft.

We notice that these functions have horizontal asymptotes but physically these do not correspond to arbitrary height Lincoln Block® walls. However they do serve as underestimates of the maximum point loading allowed for these walls, and are listed below. Due note, this model is focused on the determination force response of the splines and block alone. Thus, these values must be compared with the maximum loads of the other components in the continuous load path from roof to foundation for the specific project.

Length (ft)	Max Load (lb) $\delta = 1/12$	Max Load (lb) $\delta = 0$
15	1248	1172
25	2065	1939
33	2718	2552
35	2881	2705
45	3698	3471

Table 5: Max lateral point loading for various 21 course (8.3 ft) high walls reinforced and not.

To complete this section we consider a fixed height wall taken to be 21 courses high. This yields the following chart, useful for choosing an appropriate spline density for a given length of wall.

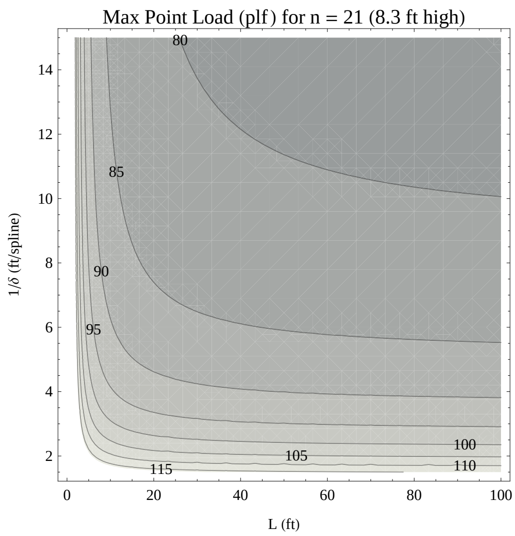

Figure 16: Max point loading for a 8.3 ft high (21 course) walls.

3.4.2 Uniform Loading (Wind)

Next we consider the maximum course drift Δx given a fixed wind pressure. Again mirroring the Nickerson Engineering preliminary calculations, we follow the assumption of a fixed pressure of

10 psf. This pressure is slightly in excess of the predicted pressure given site exposure to 110 mph gusts[7]. Applied to a 33 ft long adjoining wall and split between identical shear walls, this wind pressure induces a linear pressure of $Q = 165\ lbf/ft = 13.75\ lbf/in$ on each shear wall. Given this wind loading we again assume 12 ft spline spacing to plot the maximum course drift as a function of number of courses for various wall lengths.

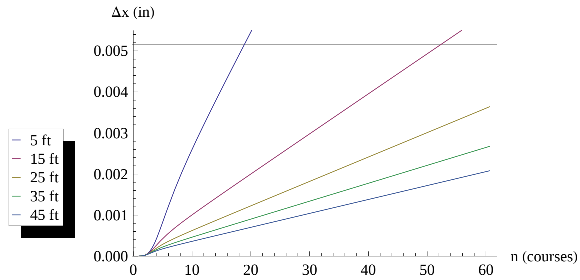

Figure 17: Max expected course drift for various unperforated and reinforced $\delta = (12\ ft)^{-1}$ shear walls resisting a design wind load of 10 psf acting upon a 33 ft adjacent windward wall.

In the above figure 18, we see that this wind load results in threshold course drift for for 5 ft wide 18 course, and 15 ft wide 52 course walls. Also observe that a 21 course (8.3 ft) high wall, 25 ft long, reinforced as described, experiences a maximum course drift of 0.0013 in exposed to this wind load. This corresponds to about 20% of the maximum drift allowed.

Conversely, we determine the maximum force per unit height given $\Delta x = 0.00516\ in$. Solving we find,

$$Q = \frac{20}{3(n\cdot c)^2}\frac{c}{\Delta x}[2(U_{C,bend} + U_{C,shear} + U_{C,nails}) + \lfloor L\delta \rfloor (U_{S,bend} + U_{S,shear} + U_{S,nails}) + L{\cdot}U_B].$$

Evaluating this function for the same parameters above we find, $Q = 65.8\ lbf/in = 790\ lbf/ft$.

As the construction of any Lincoln Block® structure involves walls of different heights, lengths, and spline densities we again fix $L = 25\ ft$, $\delta = 1/12$, or $n = 21$ to produce the following charts to aid designers. The floor function $\lfloor L\delta \rfloor$ remains replaced by $(L\delta - 1)$ and we multiply by the ratio of wall height over length to once again present values in pounds-force per lineal foot of shear wall.

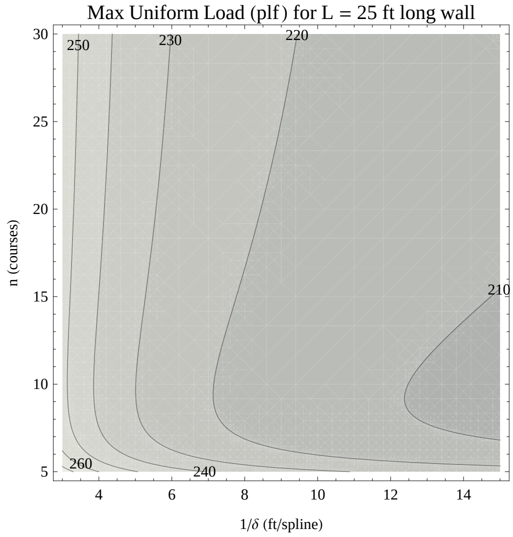

Figure 18: Max uniform load per foot of wall height for a 25 ft course wall.

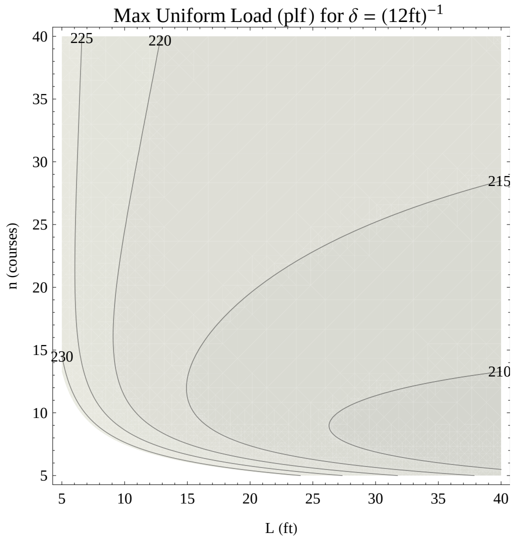

Figure 19: Max uniform load per foot of wall height reinforced with splines every 12 ft.

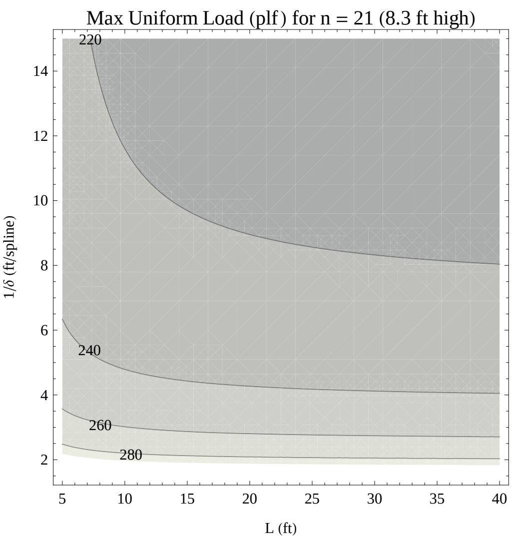

Figure 20: Max uniform load per foot of wall height for an 8.3 ft (21 course) high wall.

3.5 The 25×33 Cabin

The keen reader or design professional will note that the above subsection applies to unperforated walls. However, the design of the 25×33 cabin has two doors and eight windows. Two of these windows (on the rear wall, denoted "grid 1" in the Nickerson calculations) are supported mid-wall with *partial splines* i.e. splines that do not reach the sill plate. Conservatively we neglect these splines contribution to the wall's shear response. Below we reproduce part of the shear wall lengths table from the Nickerson report including the number of full-height splines and corner pieces along each wall. Note, we follow the previous report in neglecting shear walls shorter than 4 ft, however the application of this convention is largely arbitrary, and may cause us to underestimate δ. Also, the corner pieces appear to be double counted, however the corners stiffen both adjacent walls and in section 3.1.3 we were sure to only count the nails laterally resisting the shear loading.

Wall	Block-ft	Corners	Splines	δ^{-1}
1	20.35	2	2	10.16
2	14.75	2	8	1.84
A	14.983	2	4	3.75
B	21.92	2	2	10.96

Wall	Seismic Shear (plf)	Wind Shear (plf)
1	100	217
2	185	291
A	154	246
B	93	216

Table 6: Summary of shear resisting elements and maximum shear values for the 33 ft (walls 1,2) by 25 ft (A,B) cabin. All walls are 21 courses high and values have not been ASD adjusted.

4 Discussion

This report was dedicated to the investigation of the shear forces in the Lincoln Block® system. While an effort was made to be detailed, this is a linear analysis which required approximations and idealizations. For example, we underestimated the foam and sealant stiffness based on data availability. We also assumed the beams meet the sill plate orthogonally, and ignored the gaps between blocks of the same course. Future reports will revisit these assumptions in greater detail. However, the appropriate use of design factors should be sufficient for single story construction.

While no specific NDS codes exist limiting shear wall deflection, sufficient stiffness is required to prevent the cracking of wall treatments applied to the structure. Chapter 5 in the American Wood Council Residential Structural Design[2] guide recommends shear walls should deflect less than $l/180$ under wind and seismic loading where l is the span length in inches. Assuming no reinforcing splines ($\delta = 0$) we take the ratio of our calculated wall deflections over the prescribed maximum deflection limits to produce the below figure.

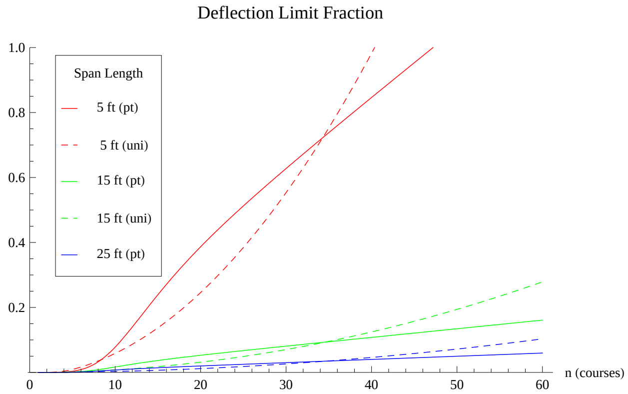

Figure 21: Deflection limit fractions relative to AWC guidelines for various un-reinforced walls.

We see that the fixed wind load from section 3.4.2 on a 40+ course, 5 ft wall is in excess of the AWC recommendations. However, for a 25 ft long, 8 ft high wall we find Lincoln Block® experiences extremely minimal deflection as described in table 6.

n (courses)	Point Defl. (in)	Limit Frac.
21	0.035	0.021
42	0.070	0.042
63	0.104	0.063
n (courses)	Uniform Defl. (in)	Limit Frac.
21	0.022	0.013
42	0.085	0.051
63	0.190	0.114

Table 7: Summary of 25 ft long wall deflections and fraction of deflection limit.

While these results are promising this model ultimately needs to be verified by experiments performed in certified facilities. While testing of fully assembled sample wall sections would be illuminating, incremental tests would inform the parameters of this model. Minimally, one should verify the force-displacement relationship for block course-to-course connections. More detailed quantitative design of such experiments however are to be released as a companion to empirical results in a later report.

Short of finite element analysis implemented in a computer aided design program, to better describe Lincoln Block® structures' behavior we propose two future studies. One into the response of walls under forces perpendicular to its face (a study in which the tolerances of the blocks' dimensions and gaps along courses will be non-negligible), and another that would build on the former two, describing the floor and roof coupling to the walls for single story and taller structures.

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